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Multi-Vantage Path Snapshot Profile for Satellite-Segment Paths: Mapping and N-Vantage Error-Exponent Scaling

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Abstract

This document defines a mapping from the Multi-Vantage Path Snapshot (MVPS) framework [I-D.melegassi-ippm-mvps-bundle] onto network paths that traverse satellite constellations and other orbital segments. The mapping reuses, without alteration, the bundle wire format, the coherence axes (C_1, C_2, C_3), the Hamiltonian H, and the Mahalanobis detection statistic D^2 of base MVPS. Two adaptations are introduced:

(1) The causal lower bound C_1 admits a vacuum propagation speed on space-segment legs and fiber refractive index on terrestrial legs.

(2) The topological coherence C_3 admits a predicted-topology component derived from publicly available Two-Line Element (TLE) sets via the SGP4 propagator [SGP4], in addition to the actual-topology component of base MVPS.

This document is informational and intentionally minimal. It states only those claims which reduce, by a finite chain of substitutions, to either (a) base MVPS theorems, or (b) classical results in special relativity and orbital mechanics. Numeric thresholds, phase-centroid values, detection latency claims, bearing-estimation accuracy, and any "X% improvement" claims are NOT made. Such results require experimental validation and are listed as Open Problems.

OPERATIONAL PREREQUISITE. The predicted-topology component C_3^pred is exercised only when per-hop satellite identity is observable at the vantage (Hypothesis H-5). No major LEO operator currently publishes such mappings; in their absence the framework degenerates to a single-axis (C_1) detector. A path-identity exposure protocol is a candidate companion specification (Open Problem OP-2).

MATHEMATICAL CORE. Under conditional independence of vantages, the joint missed-detection error exponent equals the sum of per-vantage Kullback-Leibler divergences (Appendix A; Stein’s Lemma plus KL chain rule). The non-trivial multi-vantage gain is information-theoretic: for attack classes where a single vantage has zero divergence, only the joint detector achieves beta below 1 - alpha.

The document is intended for use by network operators of LEO ground segments, by national telecommunications regulators considering independent verification of foreign-operated constellation traffic over their territory, and by the IETF community.

Status of This Memo

Copyright Notice

Table of Contents

1. Introduction

Low-Earth Orbit (LEO) constellations carry a growing share of Internet traffic. Their distinguishing properties are:

- Inter-satellite links (ISLs), where applicable, propagate at vacuum speed-of-light c, in contrast to terrestrial fiber legs which propagate at approximately 2/3 c.

- The ISL topology graph at any future time t is computable in advance from publicly available Two-Line Element (TLE) data and a standard propagator [SGP4].

PROBLEM. Single-probe measurement (e.g., a single traceroute or ping) cannot, in general, distinguish among:

(a) a Kepler-predicted ISL topology change ("orbital handover"),
(b) an unrelated routing change in the ground segment, and
(c) an attempt to manipulate the path through an unauthorized intermediary.

Cases (a)-(c) can produce indistinguishable RTT signatures at a single vantage. Multi-vantage measurement, plus the predicted-topology trajectory derivable from TLE, is sufficient to break this degeneracy in principle (Theorem 5).

APPROACH. This document defines a MAPPING from base MVPS [I-D.melegassi-ippm-mvps-bundle] onto orbital paths. The mapping is minimal and reuses the existing wire format, coherence axes, Hamiltonian, and detection statistic. The two adaptations are: (1) a mixed-medium causal lower bound for C_1, and (2) a predicted-topology component for C_3.

SCOPE. This document does NOT redefine MVPS, does NOT introduce new wire fields, does NOT define a new detection algorithm, and does NOT make numerical claims about detection latency, false-alarm rates, or improvement factors over single-probe methods. Such claims require experimental validation and are listed as Open Problems (Section 9).

2. Terminology

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "NOT RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in BCP 14 [RFC2119] [RFC8174].

LEO Low Earth Orbit, typically 340-2000 km altitude.

ISL Inter-Satellite Link. Direct optical or RF link between two satellites, bypassing the ground segment.

TLE Two-Line Element set. Standard format for encoding satellite orbital parameters [TLE-FORMAT].

SGP4 Simplified General Perturbations 4, a deterministic orbital propagator [SGP4].

GV Ground Vantage. An MVPS vantage implemented as a ground station capable of measuring path properties to a target through the satellite segment.

T_orb Orbital period of a satellite shell, derived from TLE.

T_w MVPS coordination window (inherited from base MVPS).

c Speed of light in vacuum, 299,792,458 m/s.

c_f Effective speed of light in single-mode optical fiber, approximately 2 * 10^8 m/s. Vendor-dependent.

E_v The directed edge set observed by vantage v: ordered pairs of path-identity tokens (e.g., satellite IDs) along the path. Identity tokens are vantage-side observable; see Section 3.2.

E_pred(t) The directed edge set of the Kepler-optimal path between a given source-destination pair at time t, computed from

TLE via SGP4 and an ISL-graph rule (Section 5.2).

C_1, C_2, C_3   MVPS coherence axes, inherited from base MVPS.

H   Hamiltonian, inherited from base MVPS:
          H = -(log C_1 + log C_2 + log C_3).

D^2   Mahalanobis statistic on C(t), inherited from base MVPS.

3. Mapping from MVPS-Bundle to Orbital Segments

3.1. Vantage types

Two vantage types are defined:

GV (Ground Vantage):
A ground station with traceroute-class measurement capability toward a target via the satellite segment. Requirements (REQUIRED for the math to apply):

- Known geodetic coordinates (WGS-84) to within +/- 10 m.
- UTC-synchronized clock with skew tau_clk much smaller than the coordination window T_w (Section 4, H-3).
- Ability to participate in the MVPS bundle coordination protocol of [I-D.melegassi-ippm-mvps-bundle].

RV (Relay Vantage, OPTIONAL):
A station with operator-side telemetry access (ISL neighbor tables, ground-routing tables). When available, an RV provides ground truth for the topology graph and reduces dependence on externally observable path identity. RVs are not required for the framework; they strengthen it when present.

The minimum deployment is N = 3 GVs at geographically separated sites. N = 3 is the minimum imposed by base MVPS Operational Contract OC1 (geometric-median Byzantine bound).

3.2. Path objects and path identity

The MVPS path fingerprint of base MVPS is a finite-alphabet identifier of the path observed by a vantage within a coordination window. In the orbital setting, the natural identifier is

    F = (id_entry, hop_1, hop_2, ..., hop_k, id_exit, GW_ASN)

where each id_i is a satellite identifier (e.g., NORAD catalog number) when exposed by the path-discovery mechanism in use, and GW_ASN is the autonomous system number of the ground gateway when determinable.

PATH IDENTITY OBSERVABILITY (REQUIRED for E_v to be non-trivial). The fingerprint F is meaningful only if the underlying mechanism exposes per-hop identity tokens at the vantage. For some constellations and configurations this is currently not the case (Section 10). When path identity is not observable at the vantage, the C_3 axis collapses to a coarse signal and the mathematics of T-6 is not exercised; the framework degenerates to a single-axis (C_1) detector.

3.3. Coordination window T_w

T_w is inherited from base MVPS unchanged. Implementations SHOULD set T_w small relative to the expected ISL topology change rate for the constellation under study, so that within a single window the topology is approximately stable. No specific numeric value is mandated by this document.

4. Hypotheses on which the Mathematics Depends

The theorems in Section 5 are conditional on the following hypotheses. Implementations and reviewers should assess whether each hypothesis is satisfied in the deployment under study.

H-1 TLE accuracy.
The TLE in use, after SGP4 propagation to the measurement time, gives satellite positions whose error is small enough that the predicted topology graph $G_{pred}(t)$ (Section 5.2) differs from the true topology only at link transitions within a bounded uncertainty window. Civilian TLE at epoch age <= 24 h is commonly cited as having sub-kilometer positional accuracy; this is an empirical assumption, not a theorem.

H-2 Truthful path-fingerprint reporting.
Each vantage reports the (RTT, F, E_v) it actually observes, without intentional alteration. Adversarial vantages are handled by the geometric-median bound of base MVPS Theorem 9.

H-3 Vantage clock synchronization.
The pairwise UTC clock skew across vantages, τ_{clk}, satisfies τ_{clk} much smaller than T_w. This is REQUIRED for the joint causal bound (T-1) to be meaningful and for Mahalanobis stationarity (T-4).

H-4 Distributional assumption for D^2.
Base MVPS Theorem 2 (χ^2 null) holds under a Gaussian null on C(t). In the orbital setting, C(t) is multimodal due to handover periodicity, so this hypothesis is NOT expected to hold globally. Production deployments MUST use empirical FAR calibration (base MVPS Operational Contract OC3) over a baseline that excludes predicted handover windows.

H-5 Path-identity exposure.
Theorem 6 (predicted-topology component) applies only when per-hop identity is observable at the vantage (Section 3.2). Where it is not, the framework reduces to base MVPS without the orbital extension.

5. Theorems

This section states the mathematical claims of this document. Each is either trivially derivable, inherited from a base MVPS theorem, or follows from classical relativity / orbital mechanics.

5.1. T-1: Vacuum lower bound for RTT

STATEMENT. Consider a path P between a source vantage and a destination, decomposed into legs i = 1..L, each with effective one-way propagation speed c_i, where $c_i \le c$ (special relativity). Then the round-trip time satisfies

$$RTT_{min}(P) = 2 \cdot \sum_{i=1}^{L} (d_i / c_i) \le RTT_{obs}(P)$$

in the absence of clock-skew error.

PROOF SKETCH. By special relativity, the one-way time for any signal traversing a leg of length d_i is at least d_i / c_i. Summing over legs and doubling for round trip yields the bound.

The bound is realized only in the absence of queueing, serialization, and processing delays.

APPLICATION. In a mixed-medium path, $c_i = c$ on space-segment legs (uplink, downlink, ISL) and $c_i = c_f$ on terrestrial fiber legs. The composite RTT_min thus reflects the actual physics of the path.

5.2. T-2: TLE-determined predicted topology

STATEMENT. Given a set of TLEs in scope at time t_0 and the SGP4 propagator, the predicted ISL graph

$$G_{pred}(t) = \{ (i, j) : d_{ij}(t) < R_{ISL\_max} \text{ AND } elev_{ij}(t) > elev_{min} \}$$

is a deterministic function of the TLE set and t, for any choice of constellation-specific R_ISL_max and elev_min.

PROOF SKETCH. SGP4 is a deterministic propagator; ECI position vectors are deterministic functions of TLE and t. The Boolean edge predicate is a deterministic function of those positions. Composition of deterministic functions is deterministic.

CAVEAT. T-2 establishes computability, not accuracy. The accuracy of G_pred(t) relative to the true on-orbit topology is bounded by H-1 (TLE accuracy) and any operator-specific routing rules unknown to the vantage.

5.3. T-3: Inherited coherence axes

STATEMENT. When an MVPS bundle in the orbital setting carries per-vantage RTT, fingerprint, and edge-set, the coherence axes C_1, C_2, C_3 of base MVPS [I-D.melegassi-ippm-mvps-bundle, D1+D3+F1-F4+I1] are well-defined functionals of the bundle.

PROOF. Inheritance. No new mathematics introduced.

5.4. T-4: Inherited Mahalanobis statistic

STATEMENT. The Mahalanobis statistic $D^2(t)$ on $C(t)$ inherits the base MVPS Theorems 2 and 3'. Under the Gaussian null on $C(t)$, $D^2 \sim \chi^2(3)$. Under the non-Gaussian C in $[0,1]^3$, FAR is calibrated empirically with quantified precision under base MVPS Operational Contract OC3.

PROOF. Inheritance.

ORBITAL CAVEAT. Per H-4, the Gaussian null is NOT expected to hold globally in LEO; empirical FAR calibration over a handover-excluded baseline is REQUIRED in production.

5.5. T-5: Multi-vantage discrimination is necessary

STATEMENT. Let externally observable signature at a vantage v be the tuple (RTT_v, F_v_external) where F_v_external excludes any identity tokens not exposed at the IP layer. There exist pairs (T, T') of distinct topologies such that the externally observable signature is identical between T and T' for a single vantage v. Therefore, no measurable function of single-vantage data can distinguish T from T' at v.

PROOF SKETCH. Construct T and T' that differ only in interior ISL hops while preserving entry, exit, and total propagation delay; the externally observable signature is identical by construction. $N \geq 2$ vantages with disjoint observation geometry resolve the ambiguity by providing distinct externals for the same internal events.

5.6. T-6: Predicted-topology coherence component

STATEMENT. Define

$$C_3^{pred}(t) = \text{mean}_v \; \text{Jaccard}(E_v, E_{pred}(t))$$

where $E_{pred}(t)$ is from T-2. When the observed edge set $E_v(t)$ equals the Kepler-optimal predicted edge set $E_{pred}(t)$ exactly, $C_3^{pred}(t) = 1$. When $E_v(t)$ deviates from $E_{pred}(t)$, $C_3^{pred}(t) < 1$, with the magnitude of the drop bounded by the Jaccard distance between the two edge sets.

PROOF. Direct from the definition of Jaccard similarity and T-2.

APPLICATION. C_3^{pred} provides a discriminator independent of historical baseline: a topology change that matches Kepler predictions has $C_3^{pred} \sim 1$; a topology change that does not match has $C_3^{pred} < 1$.

5.7. T-7: Boundedness

STATEMENT. C_1, C_2, C_3 in $[0,1]$. $H = -(\log C_1 + \log C_2 + \log C_3) >= 0$.

PROOF. Direct from definitions. Inherited boundedness from base MVPS Theorem 1.

6. Coherence Axes for Orbital Paths (Definitions)

6.1. C_1 with mixed-medium causal bound

For a path P decomposed into legs (uplink, ISL_1..ISL_k, downlink, plus optional terrestrial fiber legs), the orbital-segment causal component is

$$C_1^{Einstein} = \max(0, 1 - \max(0, RTT_{min}(P) - RTT_{obs}(P)) / RTT_{obs}(P))$$

where RTT_{min} is from T-1. $C_1^{Einstein}$ equals 1 whenever $RTT_{obs} >= RTT_{min}$, which is the physically realizable regime. Values strictly less than 1 indicate a measurement inconsistency (clock skew or other instrumentation issue) and are NOT interpreted as a security signal in this document. The fingerprint-entropy component C_1^{tau} is inherited from base MVPS.

$C_1 = \min(C_1^{Einstein}, C_1^{tau})$.

6.2. C_2 over path identifier distributions

Inherited from base MVPS, with the alphabet of identifiers being the satellite-identity tokens of the path fingerprint (Section 3.2).

$C_2 = 1 - JSD_{norm}(\{p_v\}_{v=1..N})$.

6.3. C_3 with actual and predicted components

Two components are defined:

C_3^{actual} = mean Jaccard over the observed edge sets $\{E_v\}_{v=1..N}$. (base MVPS, unchanged)

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   C_3^pred    =  mean Jaccard(E_v, E_pred(t)) over vantages,
                  with E_pred(t) from T-2.   (this document)
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Implementations report both components in the bundle. A combined scalar may be formed for monitoring purposes; this document does not mandate a fixed combination. Recommended practice is to monitor the two components independently and to declare an anomaly only when BOTH drop, which is the signature implied by T-6.

6.4. D^2 detection statistic

Computed identically to base MVPS, on the vector C(t) = (C_1, C_2, C_3^actual) (or, equivalently, on (C_1, C_2, C_3^pred), or on a 4-axis extension). Implementations report the choice of axes and the calibration baseline. Per H-4, the baseline MUST exclude predicted handover windows.

7. Phase Taxonomy (Qualitative Only)

The MVPS Layer-3 phase taxonomy is extended for the orbital setting with QUALITATIVE labels. Numeric centroids for these phases are NOT specified in this document; they are an Open Problem (OP-1) and require empirical study before normative specification.

ORBITAL_HANDOVER — Predicted topology change consistent with Kepler dynamics. Defined by T-6 (C_3^pred remains close to 1 during the change).

LINK_MARGIN_DEGRADATION — Gradual change in C_1 within vacuum bounds; topology stable.

PATH_INTEGRITY_BREACH — Topology change inconsistent with Kepler predictions: C_3^actual drops AND C_3^pred drops.

INTERFERENCE_SUSPECTED — Multi-vantage informational divergence (C_2 drop) localized to a subset of vantages.

These names are CLASSIFICATION HINTS for operators; they do not imply a unique numeric region in (C_1, C_2, C_3) space.

8. Conjectures (Empirical; Not Theorems)

The following statements are EXPECTED to hold but have not been proven in this document and have not been validated by simulation or measurement. They are listed for testability.

CONJ-O-1 — After excluding predicted handover windows, the residual C(t) distribution is sufficiently close to Gaussian that base MVPS empirical FAR calibration (OC3) converges within the same n_calib bound.

CONJ-O-2 — With path-identity exposure (H-5 satisfied), Jaccard between observed and TLE-predicted edge sets remains above some operationally useful floor under nominal orbital handover.

CONJ-O-3 — N >= 3 ground vantages with separation >= 500 km provide sufficient geometric diversity to expose informational and topological divergence at C_2/C_3 levels distinguishable from measurement noise.

CONJ-O-4 The bearing of an interfering source CAN be estimated from the spatial pattern of C_2 residuals across N vantages, given a beam-pattern model. No estimator is proposed in this document.

9. Open Problems

OP-1 Reference simulation. Implement a minimal simulator using (a) public TLE feeds, (b) an SGP4 propagator, (c) a constellation-specific ISL-graph rule, (d) >= 3 synthetic ground vantages, (e) injected perturbations of three classes (handover-only, interference-like, breach-like). Measure empirical FAR and characterize C(t) distribution.

OP-2 Path-identity exposure protocol. Define a JSON-signed IP-to-satellite-identity mapping that LEO operators MAY publish to enable interoperable monitoring. This is a candidate companion I-D.

OP-3 Bearing-estimation derivation. Derive a bearing estimator from a beam-pattern model. Quantify uncertainty.

OP-4 Multimodal C(t) distribution. Characterize the distribution of C(t) once handover periodicity is decoupled. Determine when distribution-free quantile methods dominate parametric chi^2 approximations.

OP-5 Timing-attack model. If a credible attack on satellite timing oscillators is to be detected, derive the observable signature on (C_1, C_2, C_3) from the underlying physics. This document does not address GNSS spoofing.

OP-6 Relay-network extension. Extend the framework to long-baseline relay networks (e.g., AU-scale propagation, hour-scale RTTs). The algebra is unchanged; the constants and TLE replacements differ.

OP-7 TLE integrity. Cross-validate TLE feeds across independent publishers to defend against maliciously crafted TLEs that could mask a topology breach by inducing a matching predicted topology.

10. Limitations

This document explicitly notes the following limitations.

L-1 No experimental validation has been performed for this extension. All theorems are mathematical or inherited; empirical claims are deferred to OP-1.

L-2 Path-identity exposure is required for the orbital-specific contributions (T-6, C_3^pred) to be exercised. As of this document’s date, several major LEO constellations do not publish stable per-satellite identity mappings. In such cases the framework reduces to base MVPS; the orbital adaptation is dormant.

L-3 TLE feeds for civilian use are public but unsigned. An adversary capable of injecting falsified TLE could mask a topology breach (OP-7).

L-4 No bearing estimator is proposed in this document (Section 8, CONJ-O-4; Section 9, OP-3). Claims of interference geolocation are out of scope.

L-5 No detection-latency or false-alarm-rate numbers are claimed. Such numbers depend on the specific deployment and on the resolution of OP-1.

L-6 No claim is made about NASA, SpaceX, or other specific operators. Where any deployment is mentioned, it is as a candidate use case, not as an existing implementation.

11. Sovereign Monitoring as a Use Case

The framework is consistent with passive multi-station monitoring by a national regulator. An N >= 3 ground-vantage deployment over national territory, using public TLE and base MVPS bundles, can produce signed observations of (C_1, C_2, C_3, D^2) trajectories.

The mathematics permits, but does not guarantee, that such a deployment can independently verify whether traffic over national territory follows the routing claimed by the constellation operator. Whether this verification succeeds in any given deployment depends on Hypothesis H-5 (path-identity exposure) and on Limitation L-2.

This document makes no policy or legal claim. Operators of any sovereign deployment SHOULD verify compliance with national and international telecommunications law and ITU Radio Regulations.

12. Security Considerations

This document defines a monitoring framework. No new wire format is introduced; security considerations of base MVPS apply unchanged. Additional considerations specific to the orbital setting:

Adversarial-evasion mitigation. An adversary aware of this framework might attempt to mimic an orbital handover signature in order to evade T-6 detection. Mimicry requires the adversary to compute, in real time, a topology change that matches both E_pred(t) and timing predicted by SGP4 from the public TLE feed. The cost of such mimicry depends on the adversary’s compute budget and on the integrity of the TLE feed (OP-7).

TLE integrity. See L-3 and OP-7.

Confidentiality. MVPS bundles inherit confidentiality considerations from base MVPS; no orbital-specific exposure is introduced.

Bundles SHOULD be cryptographically signed with vantage identity keys when used for sovereign monitoring (Section 11), to support forensic use.

13. IANA Considerations

This document requests no IANA actions.

A future companion document might propose an IANA registry for constellation operator identifiers (mapping operator name to ASN range, TLE feed URL, and identity-exposure protocol version). That registry is out of scope for this document.

14. References

14.1. Normative References

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14.2. Informative References

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[Cover-Thomas-2006] Cover, T. M. and Thomas, J. A., "Elements of Information Theory", 2nd edition, Wiley-Interscience, 2006. (Stein’s Lemma: Theorem 11.8.1, Chapter 11.)

[CAIB-2003] Columbia Accident Investigation Board, "Report Volume I", NASA, August 2003. Public domain. (Cited for the class-of-gap precedent in multi-source orbital observation.)

[NASA-OIG-IG-23-016] NASA Office of Inspector General, "Audit of NASA’s Deep Space Network", Report IG-23-016, 12 July 2023. Public domain. In particular Recommendation R-4 (utilization of commercial and international partner networks as DSN backup), which is the operational opening for the framework defined in the present document.

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[RFC9198] Ciavattone, L., Morton, A., Linsner, M., and J. Seitz, "Advanced Unidirectional Route Assessment (AURA)", RFC 9198, May 2022.

Appendix A. N-Vantage Information Advantage (Classical Result)

This appendix records, for completeness, the classical detection-theoretic bound that motivates N >= 3 multi-vantage operation in base MVPS and inherited by this document. The theorem and its proof are taken from [Cover-Thomas-2006], Theorem 11.8.1 (Stein’s Lemma), with the standard extension to independent observers via the chain rule for Kullback-Leibler divergence. No new mathematics is introduced.

A.1. Setting

Consider a binary hypothesis test on the state of an orbital path:

H_0 : the path is in nominal state.
H_1 : the path is in an anomalous state (handover-but-not-predicted, interference, or path-integrity breach; see Section 7).

Each ground vantage v in $\{1, ..., N\}$ produces an observation X_v drawn from P_v^0 under H_0 and from P_v^1 under H_1.

Assumptions:

- A1. $X_1, ..., X_N$ are conditionally independent given the hypothesis. This corresponds to instrumentally and geographically independent vantages.
- A2. For each v, the Kullback-Leibler divergence $D_v := KL(P_v^1 \| P_v^0)$ is finite and positive.

A.2. Stein's Lemma (single observer)

For a single vantage v, with n independent samples drawn under H_1 and any test sequence whose Type-I error is bounded by $\alpha < 1$, the optimal Type-II (missed-detection) error $\beta_n^v(\alpha)$ satisfies:

$$-(1/n) \cdot \log \beta_n^v(\alpha) \longrightarrow D_v \qquad (n \to \infty).$$

([Cover-Thomas-2006], Theorem 11.8.1.)

A.3. N-vantage extension

By A1, the joint distribution under hypothesis H_k is $P_{joint}^k = P_1^k \times P_2^k \times ... \times P_N^k$. By the chain rule for KL divergence on independent components:

$$KL(P_{joint}^1 \| P_{joint}^0) = \sum_{v=1..N} D_v.$$

Applying Stein's Lemma to the joint observation:

$$\beta_n^{joint}(\alpha) = \exp(-n \cdot \sum_{v=1..N} D_v + o(n)).$$

A.4. THEOREM A-1. N-Vantage Information Advantage

Under A1-A2, the asymptotic missed-detection error exponent of the joint N-vantage test exceeds the error exponent of the best single-vantage test:

$$E_N := \sum_{v=1..N} D_v \geq \max_v D_v =: E_1,$$

with strict inequality whenever $D_v > 0$ for at least two vantages.

PROOF. Sum over non-negative reals dominates max over the same set; strict inequality follows when at least two summands are positive.

A.5. Operational interpretation

Theorem A-1 is the mathematical content of the requirement $N \geq 3$ imposed by base MVPS Operational Contract OC1. It does NOT, by itself, predict the empirical error exponent achieved by any specific deployment, which depends on per-vantage D_v values (Conjecture CONJ-O-3 in Section 8). It DOES guarantee that every additional independent vantage strictly improves the missed-detection bound at fixed false-alarm rate.

This is the property by which radar arrays, GNSS spoofing detectors, and seismic event-location networks justify their multi-observer architectures. The orbital-coherence framework inherits this property without modification.

A.6. Limitations

Theorem A-1 is asymptotic. It assumes conditional independence (A1). Vantages that share systematic biases (e.g., a common TLE feed compromised under OP-7) violate A1 and the bound does not apply directly to the compromised component. Theorem A-1 is stated here for the orbital case for completeness; its proof is not original to this document.

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